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MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS
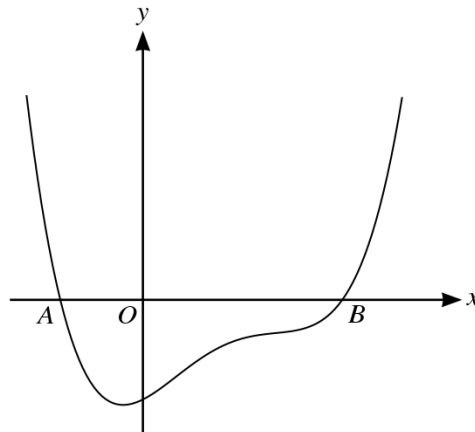
Chapter 6

Numerical solution of equations

189. 9709_w20_qp_21 Q: 5

The sequence of values given by the iterative formula $x_{n+1} = \dfrac{6 + 8x_n}{8 + x_n^2}$ with initial value $x_1 = 2$ converges to α.

(a) Use the iterative formula to find the value of α correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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(b) State an equation satisfied by α and hence determine the exact value of α. [2]

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190. 9709_w20_qp_22 Q: 7

A curve has equation $y = f(x)$ where $f(x) = x^4 - 5x^3 + 6x^2 + 5x - 15$. As shown in the diagram, the curve crosses the x-axis at the points A and B with coordinates $(a, 0)$ and $(b, 0)$ respectively.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. [2]

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(b) By first finding the quotient when $f(x)$ is divided by $(x - 3)$, show that

$$a = -\sqrt{\frac{5}{2 - a}}.$$ [5]

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(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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191. 9709_m19_qp_22 Q: 5

The equation of a curve is $y = \dfrac{e^{2x}}{4x + 1}$ and the point P on the curve has y-coordinate 10.

(i) Show that the x-coordinate of P satisfies the equation $x = \frac{1}{2}\ln(40x + 10)$. [2]

(ii) Use the iterative formula $x_{n+1} = \frac{1}{2}\ln(40x_n + 10)$ with $x_1 = 2.3$ to find the x-coordinate of P correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

(iii) Find the gradient of the curve at P, giving the answer correct to 3 significant figures. [4]

192. 9709_s19_qp_21 Q: 6

The diagram shows the curve with equation $y = \dfrac{8 + x^3}{2 - 5x}$. The maximum point is denoted by M.

(i) Find an expression for $\dfrac{dy}{dx}$ and determine the gradient of the curve at the point where the curve crosses the x-axis. [4]

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(ii) Show that the x-coordinate of the point M satisfies the equation $x = \sqrt{(0.6x + 4x^{-1})}$.   [2]

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(iii) Use an iterative formula, based on the equation in part **(ii)**, to find the x-coordinate of M correct to 3 significant figures. Give the result of each iteration to 5 significant figures.   [3]

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The diagram shows the curve with parametric equations

$$x = 3t - 6e^{-2t}, \quad y = 4t^2 e^{-t},$$

for $0 \leqslant t \leqslant 2$. At the point P on the curve, the y-coordinate is 1.

(i) Show that the value of t at the point P satisfies the equation $t = \frac{1}{2}e^{\frac{1}{2}t}$. [2]

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(ii) Use the iterative formula $t_{n+1} = \frac{1}{2}e^{\frac{1}{2}t_n}$ with $t_1 = 0.7$ to find the value of t at P correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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(iii) Find the gradient of the curve at *P*, giving the answer correct to 2 significant figures.   [5]

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194. 9709_w19_qp_21   Q: 5

It is given that $\int_0^a (3x^2 + 4\cos 2x - \sin x)\,dx = 2$, where a is a constant.

(i) Show that $a = \sqrt[3]{(3 - 2\sin 2a - \cos a)}$. [4]

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(ii) Using the equation in part **(i)**, show by calculation that $0.5 < a < 0.75$. [2]

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(iii) Use an iterative formula, based on the equation in part **(i)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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195. 9709_w19_qp_22 Q: 4

The sequence $x_1, x_2, x_3, \ldots$ defined by

$$x_1 = 1, \quad x_{n+1} = \frac{x_n}{\ln(2x_n)}$$

converges to the value α.

(i) Use the iterative formula to find the value of α correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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(ii) State an equation satisfied by α and hence determine the exact value of α. [2]

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196. 9709_m18_qp_22 Q: 5

It is given that $\int_{-a}^{2a} 4e^{-2x}\,dx = 25$, where a is a positive constant.

(i) Show that $a = \frac{1}{2}\ln(12.5 + e^{-4a})$. [4]

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(ii) Use the equation in part **(i)** to show by calculation that $1.0 < a < 1.5$. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 4 significant figures. Show the result of each iteration to 6 significant figures. [3]

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197. 9709_s18_qp_21 Q: 4


The diagram shows the curve with equation $y = \dfrac{5 \ln x}{2x + 1}$. The curve crosses the x-axis at the point P and has a maximum point M.

(i) Find the gradient of the curve at the point P. [3]

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(ii) Show that the x-coordinate of the point M satisfies the equation $x = \dfrac{x + 0.5}{\ln x}$. [2]

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(iii) Use an iterative formula based on the equation in part **(ii)** to find the x-coordinate of M correct to 4 significant figures. Show the result of each iteration to 6 significant figures. [3]

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198. 9709_s18_qp_22   Q: 6

It is given that $\int_0^a \left(1 + e^{\frac{1}{2}x}\right)^2 dx = 10$, where a is a positive constant.

(i) Show that $a = 2\ln\left(\dfrac{15 - a}{4 + e^{\frac{1}{2}a}}\right)$. [6]

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(ii) Use the equation in part **(i)** to show by calculation that $1.5 < a < 1.6$. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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199. 9709_w18_qp_21 Q: 4

The diagram shows the curve with equation

$$y = x^4 + 2x^3 + 2x^2 - 12x - 32.$$

The curve crosses the x-axis at points with coordinates $(\alpha, 0)$ and $(\beta, 0)$.

(i) Use the factor theorem to show that $(x + 2)$ is a factor of

$$x^4 + 2x^3 + 2x^2 - 12x - 32.$$ [2]

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(ii) Show that β satisfies an equation of the form $x = \sqrt[3]{(p + qx)}$, and state the values of p and q. [3]

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(iii) Use an iterative formula based on the equation in part **(ii)** to find the value of β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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200. 9709_w18_qp_22 Q: 5

The curve with equation

$$y = 5e^{2x} - 8x^2 - 20$$

crosses the x-axis at only one point. This point has coordinates $(p, 0)$.

(i) Show that p satisfies the equation $x = \frac{1}{2}\ln(1.6x^2 + 4)$. [2]

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(ii) Show by calculation that $0.75 < p < 0.85$. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 5 significant figures. Give the result of each iteration to 7 significant figures. [3]

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(iv) Find the gradient of the curve at the point $(p, 0)$. [3]

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201. 9709_m17_qp_22 Q: 5

It is given that a is a positive constant such that

$$\int_0^a (1 + 2x + 3e^{3x})\,dx = 250.$$

(i) Show that $a = \frac{1}{3}\ln(251 - a - a^2)$. [5]

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(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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202. 9709_s17_qp_21 Q: 4

The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^2 + x_n + 9}{(x_n + 1)^2},$$

with $x_1 = 2$, converges to α.

(i) Find the value of α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

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(ii) Determine the exact value of α. [3]

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203. 9709_s17_qp_22 Q: 3

(i) By sketching a suitable pair of graphs, show that the equation

$$x^3 = 11 - 2x$$

has exactly one real root. [2]

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(ii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(11 - 2x_n)}$$

to find the root correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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204. 9709_w17_qp_21 Q: 7
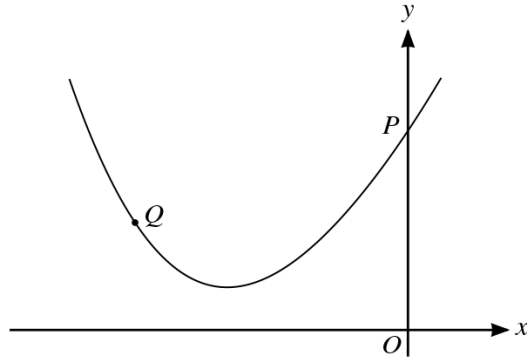
The diagram shows the curve

$$y = x^2 + 3x + 1 + 5\cos\tfrac{1}{2}x.$$

The curve crosses the y-axis at the point P and the gradient of the curve at P is m. The point Q on the curve has x-coordinate q and the gradient of the curve at Q is $-m$.

(i) Find the value of m and hence show that q satisfies the equation

$$x = a\sin\tfrac{1}{2}x + b,$$

where the values of the constants a and b are to be determined. [4]

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(ii) Show by calculation that $-4.5 < q < -4.0$. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to find the value of q correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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205. 9709_w17_qp_22 Q: 5
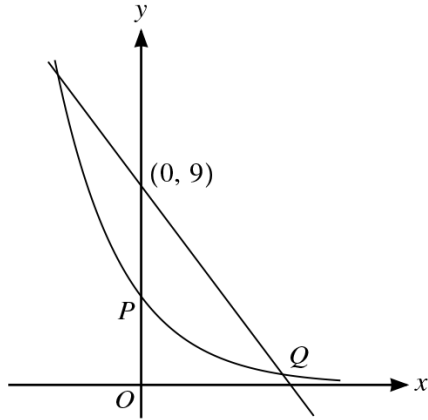
The diagram shows the curve $y = 4e^{-2x}$ and a straight line. The curve crosses the y-axis at the point P. The straight line crosses the y-axis at the point $(0, 9)$ and its gradient is equal to the gradient of the curve at P. The straight line meets the curve at two points, one of which is Q as shown.

(i) Show that the x-coordinate of Q satisfies the equation $x = \frac{9}{8} - \frac{1}{2}e^{-2x}$. [6]

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(ii) Use an iterative formula based on the equation in part **(i)** to find the x-coordinate of Q correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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206. 9709_m16_qp_22 Q: 4

The sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(\tfrac{1}{2}x_n^2 + 4x_n^{-3}\right)},$$

with initial value $x_1 = 1.5$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

**(ii)** State an equation that is satisfied by α and hence find the exact value of α. [2]
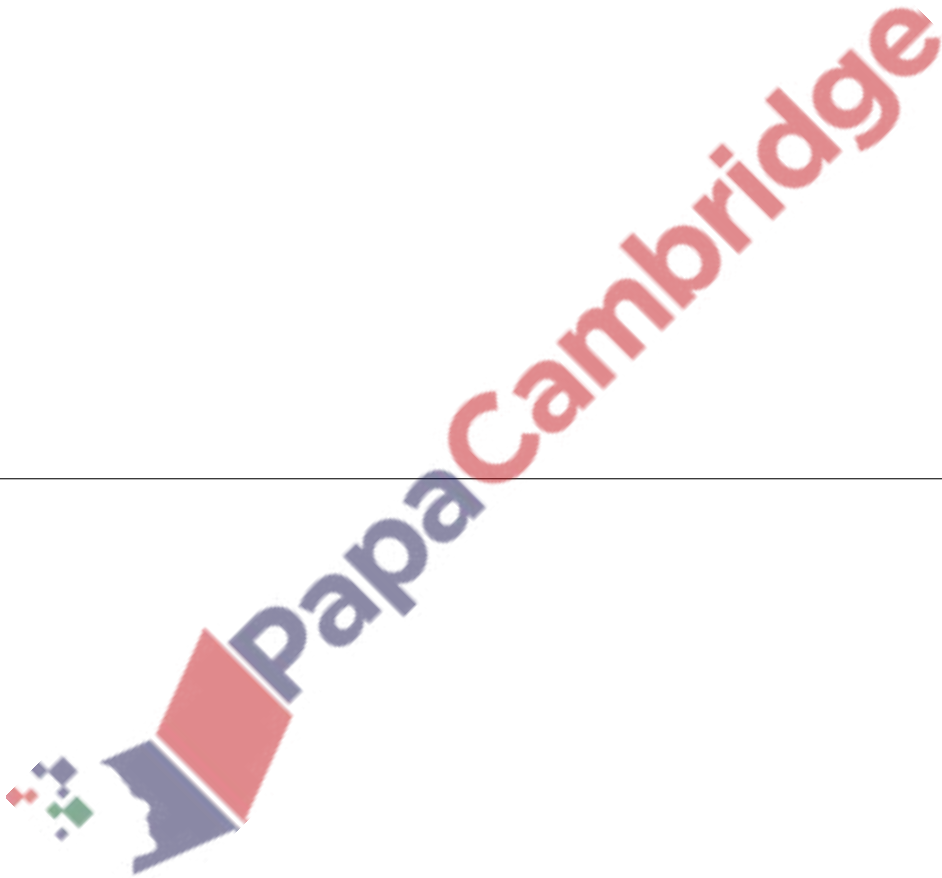
207. 9709_s16_qp_21 Q: 6

The equation of a curve is $y = \dfrac{3x^2}{x^2 + 4}$. At the point on the curve with positive x-coordinate p, the gradient of the curve is $\frac{1}{2}$.

(i) Show that $p = \sqrt{\left(\dfrac{48p - 16}{p^2 + 8} \right)}$. [5]

(ii) Show by calculation that $2 < p < 3$. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

208. 9709_s16_qp_22 Q: 5

The equation of a curve is $y = 6xe^{\frac{1}{3}x}$. At the point on the curve with x-coordinate p, the gradient of the curve is 40.

(i) Show that $p = 3\ln\left(\dfrac{20}{p+3}\right)$. [4]

(ii) Show by calculation that $3.3 < p < 3.5$. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]
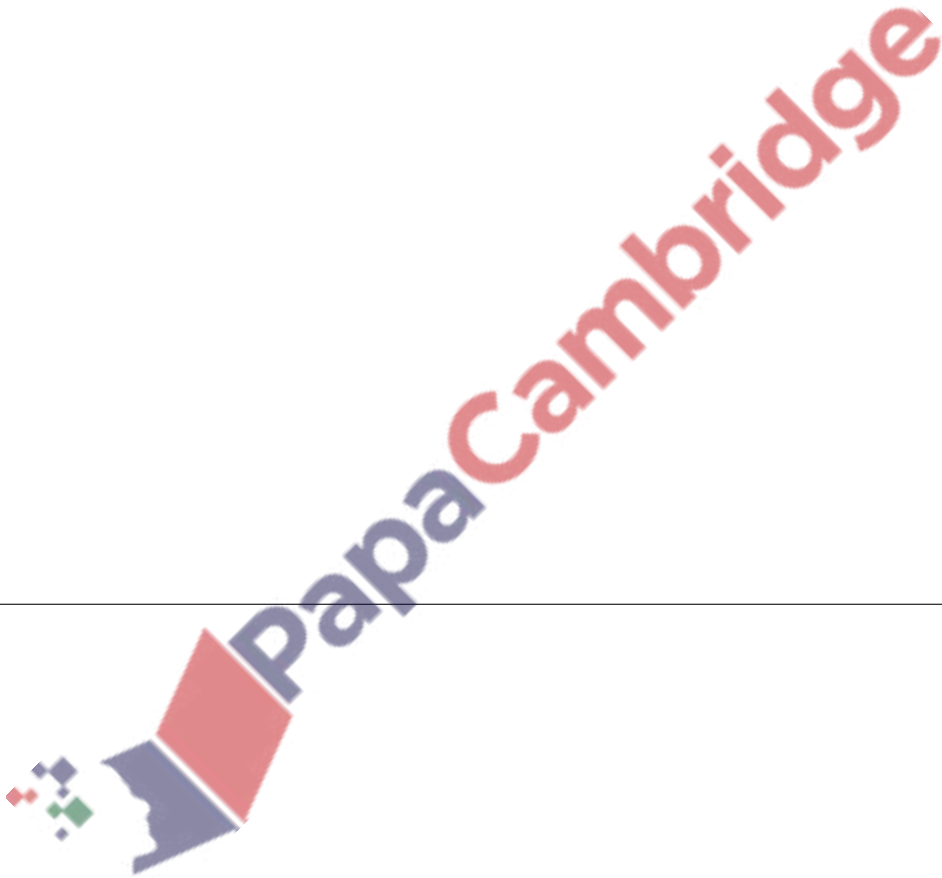
209. 9709_w16_qp_21 Q: 4

It is given that the positive constant a is such that

$$\int_{-a}^{a} (4e^{2x} + 5)\, dx = 100.$$

(i) Show that $a = \frac{1}{2}\ln(50 + e^{-2a} - 5a)$. [4]

(ii) Use the iterative formula $a_{n+1} = \frac{1}{2}\ln(50 + e^{-2a_n} - 5a_n)$ to find a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]
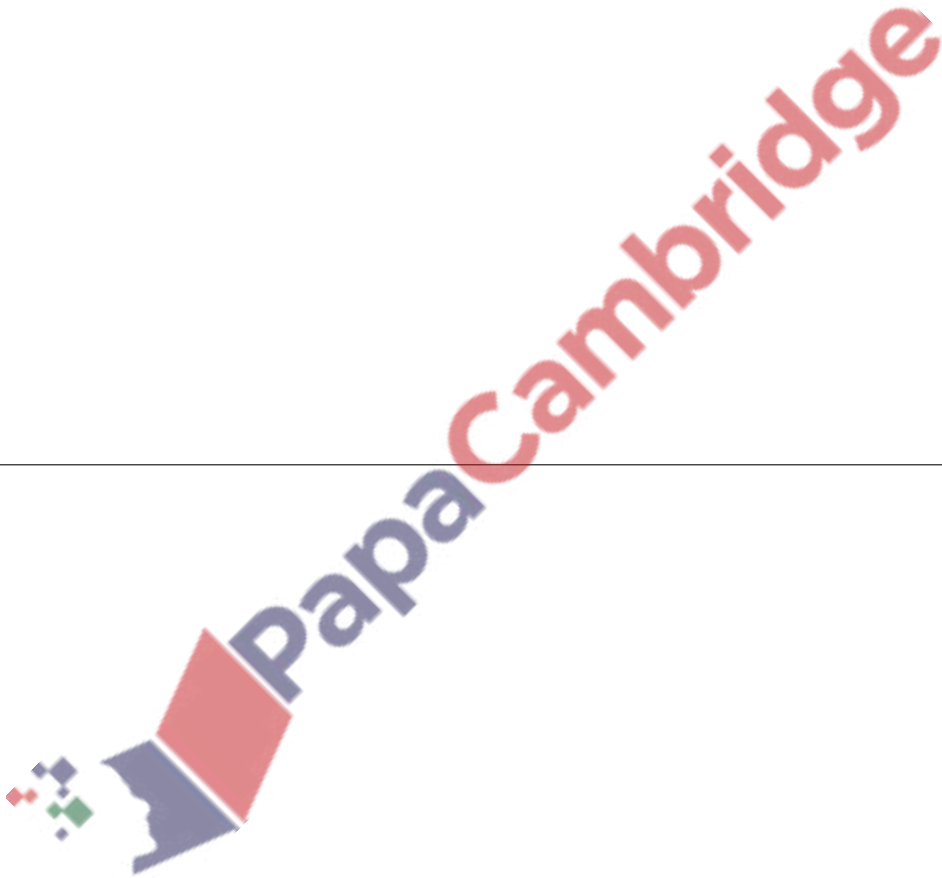
210. 9709_w16_qp_22 Q: 5
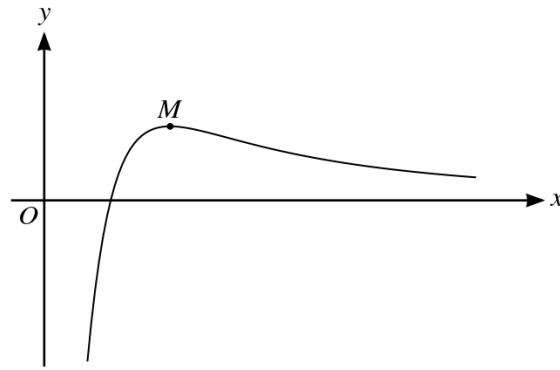
The diagram shows the curve $y = \dfrac{4\ln x}{x^2 + 1}$ and its stationary point M. The x-coordinate of M is m.

(i) Find an expression for $\dfrac{dy}{dx}$ and hence show that $m = e^{0.5(1 + m^{-2})}$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of m correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]
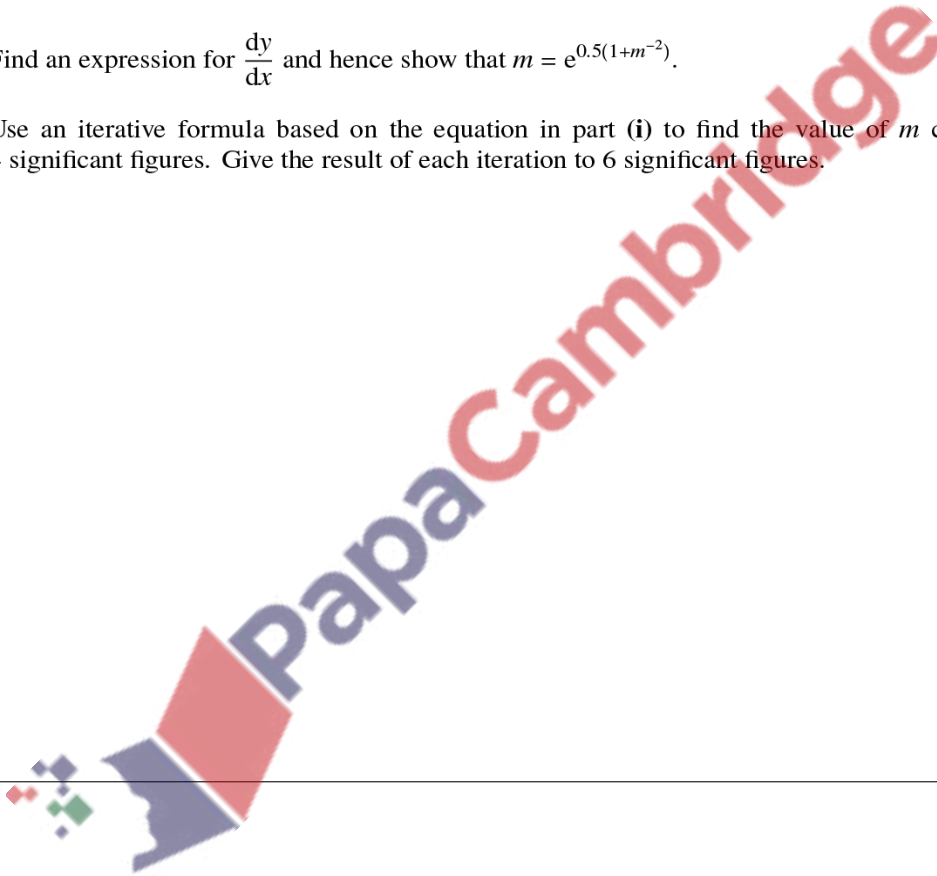
211. 9709_w16_qp_23 Q: 1

The sequence of values given by the iterative formula

$$x_{n+1} = \frac{4}{x_n^2} + \frac{2x_n}{3},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(ii) State an equation that is satisfied by α, and hence find the exact value of α. [2]
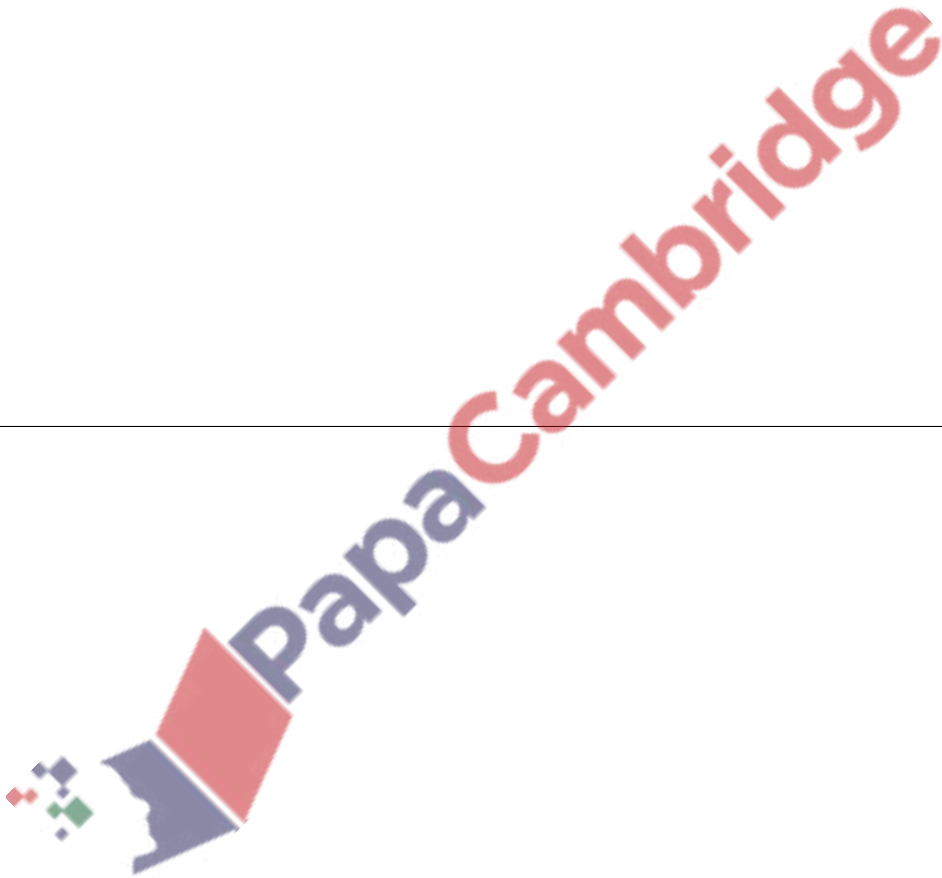
212. 9709_s15_qp_21   Q: 5

(i) Given that $\displaystyle\int_0^a (3e^{\frac{1}{2}x} + 1)\,dx = 10$, show that the positive constant a satisfies the equation

$$a = 2\ln\left(\frac{16 - a}{6}\right).$$ [5]

(ii) Use the iterative formula $a_{n+1} = 2\ln\left(\dfrac{16 - a_n}{6}\right)$ with $a_1 = 2$ to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places.   [3]
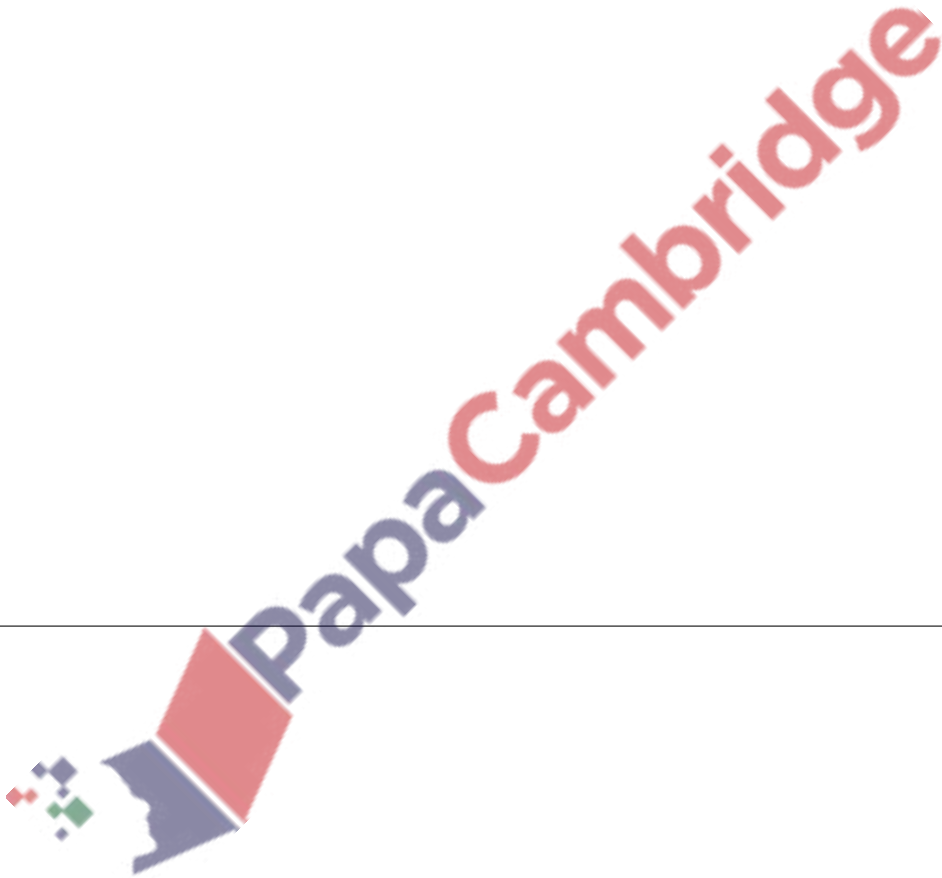
213. 9709_s15_qp_22   Q: 5

    (i) By sketching a suitable pair of graphs, show that the equation

$$|3x| = 16 - x^4$$

has two real roots.   [3]

   (ii) Use the iterative formula $x_{n+1} = \sqrt[4]{(16 - 3x_n)}$ to find one of the real roots correct to 3 decimal places. Give the result of each iteration to 5 decimal places.   [3]

  (iii) Hence find the coordinates of each of the points of intersection of the graphs $y = |3x|$ and $y = 16 - x^4$, giving your answers correct to 3 decimal places.   [2]

214. 9709_w15_qp_21 Q: 4

(i) By sketching a suitable pair of graphs, show that the equation

$$\ln x = 4 - \tfrac{1}{2}x$$

has exactly one real root, α.   [2]

(ii) Verify by calculation that $4.5 < \alpha < 5.0$.   [2]

(iii) Use the iterative formula $x_{n+1} = 8 - 2\ln x_n$ to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places.   [3]
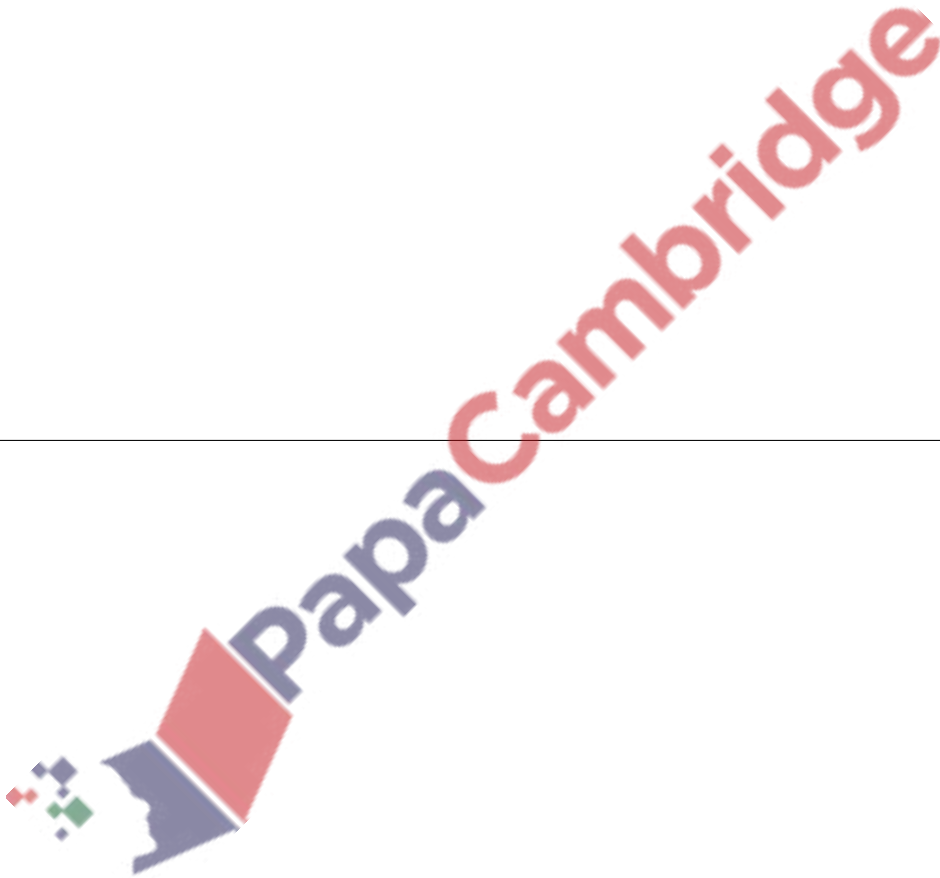
215. 9709_w15_qp_22 Q: 2

The sequence of values given by the iterative formula

$$x_{n+1} = 2 + \frac{4}{x_n^2 + 2x_n + 4},$$

with initial value $x_1 = 2$, converges to α.

(i) Determine the value of α correct to 3 decimal places, giving the result of each iteration to 5 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]
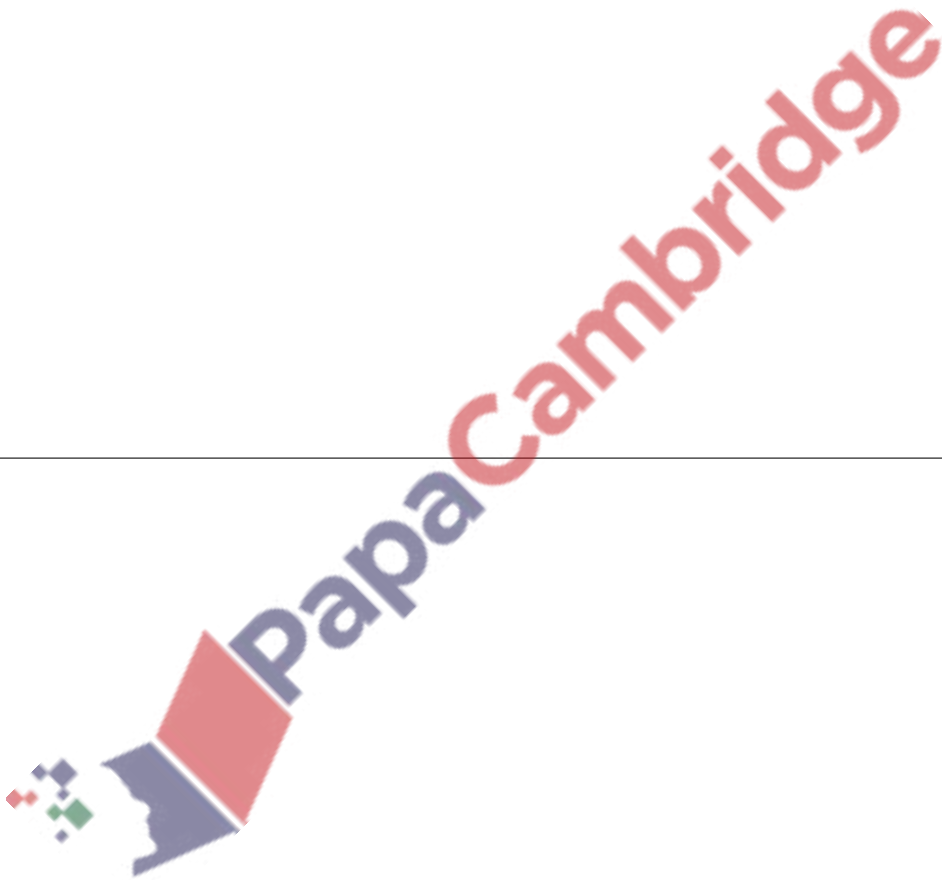
216. 9709_w15_qp_23 Q: 5

It is given that $\displaystyle\int_0^a (3e^{3x} + 5e^x)\,dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{3}\ln(106 - 5e^a)$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]